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Quarkonium chromo-polarizability from the decays $J/\psi(\Upsilon) \to \pi\pi\ell^+\ell^-$

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Abstract

It is pointed out that the diagonal amplitude of the $E1-E1$ chromo-electric interaction with soft gluon fields (chromo-polarizability) can be measured directly for the J/ψ and Υ resonances in the decays $J/\psi \to \pi\pi\ell^+\ell^-$ and $\Upsilon \to \pi\pi\ell^+\ell^-$ with soft pions. For the J/ψ this amplitude is often discussed in connection with the J/ψ interaction with nuclear matter, while for the Υ the chromo-polarizability enters the estimates of the non-perturbative mass shift of the resonance relevant to precision determination of the b quark mass from the Υ mass.

It is realistic to expect that a very high statistics data on heavy quarkonia will become available in a foreseeable future. In particular the plans of the CLEO-c experiment[1] include acquiring about 10^9 events at the J/ψ resonance and, given the capabilities of the B factories, a great increase in the statistics of the Υ resonances is also quite possible. With such amount of data quite rare processes can be studied, which can help in resolving some of the old-standing problems in dynamics of heavy quarkonia. The purpose of the present letter is to point out that a study of the decays $J/\psi \to \pi\pi\ell^+\ell^-$ and $\Upsilon \to \pi\pi\ell^+\ell^-$ with soft pions would allow to measure the gluon polarizability of the respective quarkonium resonance, which determines the strength of the quarkonium interaction with soft gluon field, and which thus far is only guessed theoretically.

The chromo-polarizability α of a state of quarkonium can be defined (in complete analogy with the usual polarizability of atoms) through the effective Hamiltonian of the quarkonium state interaction with a soft chromo-electric field $\vec{E}^a$:

$$H_{eff} = -\frac{1}{2}\,\alpha\,\vec{E}^a \cdot \vec{E}^a \; . \tag{1}$$

This effective interaction arises in the second order in the leading $E1$ chromo-electric dipole term in the multipole expansion[2, 3] of the interaction of a heavy quarkonium with the gluon field:

$$H_{E1} = -\frac{1}{2}\xi^a\,\vec{r} \cdot \vec{E}^a(0) \; , \tag{2}$$

where $\xi^a = t_1^a - t_2^a$ is the difference of the color generators acting on the quark and antiquark (e.g. $t_1^a = \lambda^a/2$ with λ^a being the Gell-Mann matrices), and $\vec{r}$ is the vector for relative position of the quark and the antiquark. (In the normalization used throughout this paper the QCD coupling g is included in the definition of the field, so that e.g. the gluon field Lagrangean reads as $L = -(F^a_{\mu\nu})^2/(4g^2)$.) Thus for a colorless S wave state the chromo-polarizability α is given by the diagonal matrix element:

$$\alpha = \frac{1}{48}\,\langle S|\,\xi^a\,r_i\,G_A\,r_i\,\xi^a|S\rangle \; , \tag{3}$$

where G_A is the Green's function for a heavy quark pair in color-octet (adjoint) state. A theoretical understanding of such matrix element is at least highly model dependent at present due to its sensitivity to the wave function of the quarkonium and due to presently unknown propagator G_A of a colored quark pair.

The chromo-polarizability of J/ψ: $\alpha_{J/\psi}$, enters as an important parameter in analyses of the J/ψ interaction with nuclear matter (see e.g. in Refs. [4, 5]) and is estimated essentially

on dimensional grounds[6]. The same quantity for the Υ resonance: α_Υ, determines[3] the non-perturbative shift of the Υ mass due to the gluon vacuum condensate[7] $\langle 0|(F^a_{\mu\nu})^2|0\rangle$:

$$\delta M_\Upsilon = \frac{1}{8}\,\alpha_\Upsilon\,\langle 0|(F^a_{\mu\nu})^2|0\rangle~. \tag{4}$$

The knowledge of the non-perturbative shift of the Υ mass is important for the method of determining the b quark mass by comparing M_Υ with the perturbative QCD expression in terms of m_b for the mass of the lowest ${}^3S_1$ bound state of the quark-antiquark pair (for a detailed discussion see the review [8] and references therein). Practical estimates of this non-perturbative shift use an extrapolation down to the bottomonium of the formulas[9, 10] for asymptotically heavy quarkonium. In the latter limit the heavy quarkonium is essentially a Coulomb system, and the matrix elements of the type in eq.(3) can be found explicitly. In particular for the $1S$ state the chromo-polarizability is found[9, 10] as

$$\alpha_{1S} = \frac{78}{425}\,\frac{m_Q}{k_B^4}~, \tag{5}$$

with $k_B = 2m_Q\alpha_s(k_B)/3$ being the Bohr momentum for the heavy quarkonium. If applied to bottomonium, this formula gives $\alpha_\Upsilon \approx 1GeV^{-3}$. Using then the value of the gluon vacuum condensate[7] $\langle 0|(F^a_{\mu\nu})^2|0\rangle = \langle 0|4\pi\alpha_s\,(G^a_{\mu\nu})^2|0\rangle \approx 0.5\,GeV^4$, one estimates from eq.(4) the non-perturbative shift of the mass of Υ: $\delta M_\Upsilon \approx 50-60\,MeV$. However, in addition to the uncertainty in the value of the gluon condensate, there are at least two other uncertainties involved in this estimate: one arising from the application of the asymptotic expression (5) to bottomonium, and the other associated with possible contribution of vacuum averages of higher dimension, generally nonlocal, gluonic operators. Although it is not clear at present to what extent the latter uncertainty can be estimated, a direct measurement of the chromo-polarizability α_Υ would definitely fix at least one factor in this problem.

It should be noted that the non-diagonal amplitudes of the type in eq.(3) for the transitions between the $2S$ and $1S$ states,

$$\alpha_{1S-2S} = \frac{1}{48}\,\langle 1S|\,\xi^a\,r_i\,G_A\,r_i\,\xi^a|2S\rangle~, \tag{6}$$

can be found both in charmonium and bottomonium from the spectra and the rates of the pionic transitions $\psi' \to \pi\pi\,J/\psi$ and $\Upsilon' \to \pi\pi\,\Upsilon$. The relation arises through the fact that the dominant part of the amplitude of production of two pions by the gluonic operator $(\vec{E}^a)^2$ is determined[11] by the trace anomaly in QCD and the chiral algebra:

$$\langle \pi^+\pi^-|\,(\vec{E}^a)^2|0\rangle = \frac{8\pi^2}{b}q^2 + O(\alpha_s q_0^2) + O(m_\pi^2)~, \tag{7}$$

where $q = p_+ + p_-$ is the total 4-momentum of the pion pair (so that q_0 is the total energy of the pair), $b = 9$ is the first coefficient in the QCD beta function with three light quarks, and the subleading terms, analyzed in Ref.[12], are relatively slowly varying with q^2 in the physical region of the pionic transition. In practice these terms can be approximated by a constant C, which depends on q_0, so that the amplitude of the decay can be written according to the equations (2), (6), and (7) as

$$A(2S \to \pi^+\pi^-\, 1S) = -\frac{4\pi^2}{b}\, \alpha_{1S-2S}\, (q^2 - C) \tag{8}$$

with C being slightly different for the transitions in charmonium and in bottomonium. The observed experimental spectra of the dipion mass in the decays $\psi' \to \pi\pi\, J/\psi$ and $\Upsilon' \to \pi\pi\, \Upsilon$ agree with the form of the amplitude in eq.(8) and the with the numerical value of the constant: $C = (4.6 \pm 0.2)\, m_\pi^2$ for the transition in charmonium and $C = (3.3 \pm 0.2)\, m_\pi^2$ for the decay $\Upsilon' \to \pi\pi\, \Upsilon$ (for a discussion see e.g. the review [13]).

The total rates of the pionic transitions are calculated from eq.(8) as

$$\begin{aligned}
\Gamma(\psi' \to \pi^+\pi^-\, J/\psi) &\approx 0.30\, \frac{8\pi}{105\, b^2}\, |\alpha_{J/\psi-\psi'}|^2\, \left[M(\psi') - M(J/\psi)\right]^7 ~, \\
\Gamma(\Upsilon' \to \pi^+\pi^-\, \Upsilon) &\approx 0.36\, \frac{8\pi}{105\, b^2}\, |\alpha_{\Upsilon-\Upsilon'}|^2\, \left[M(\Upsilon') - M(\Upsilon)\right]^7 ~,
\end{aligned} \tag{9}$$

where the decimal numerical factors describe the relative suppression of the rates due to the corresponding constant term C and due to the nonzero pion mass. Thus from a comparison of these expressions with the experimental data[14] on the rates one estimates the transition chromo-polarizabilities:

$$|\alpha_{J/\psi-\psi'}| \approx 2.0\, GeV^{-3} ~, \quad |\alpha_{\Upsilon-\Upsilon'}| \approx 0.66\, GeV^{-3} ~. \tag{10}$$

These estimates of the transition matrix elements illustrate the typical values that one can expect for the diagonal chromo-polarizability in charmonium and bottomonium. It is also somewhat satisfying to notice that for the bottomonium the estimate of the diagonal chromo-polarizability from eq.(5) is in a reasonable agreement with the value of the transition term, since one generally would expect the diagonal matrix element (3) to be somewhat larger than the non-diagonal (6).

Proceeding to discussion of the decays $J/\psi \to \pi\pi\ell^+\ell^-$ and $\Upsilon \to \pi\pi\ell^+\ell^-$ we retain the notation q for the total 4-momentum of the two pions. The amplitude of such decay for a 1^3S_1

state of heavy quarkonium in the soft pion limit can be written as a sum over intermediate n^3S_1 states:

$$A(1^3S_1 \to \pi^+\pi^-\ell^+\ell^-) = \frac{1}{2} \langle \pi^+\pi^- | (\vec{E}^a)^2 | 0 \rangle \sum_{n=1} \frac{\alpha_{1S-nS}}{M(nS) - M(1S) + q_0} A(n^3S_1 \to \ell^+\ell^-) \, , \tag{11}$$

where the sum goes over the discrete states as well as the continuum. In writing this expression it is taken into account that the soft pion approximation is only valid at $q_0 \ll M(1S)$, so that any recoil of the heavy quarkonium upon emission of the pion pair can be and is neglected. In this limit the relation between q_0 and the total momentum l of the lepton pair can also be written as $M^2(1S) - l^2 = 2\, q_0\, M(1S)$.

In the chiral limit the first term (with $n = 1$) in the sum in eq.(11) dominates for soft pions, due to its singular behavior as $1/q_0$. It can be noticed however that the decay amplitude itself is not singular due to the (even faster) vanishing of the pion production amplitude (7) in the limit of soft pions. In the 'real life' the minimal practical energy q_0 is not much less than the spacing of the quarkonium levels, and the contribution of higher states mixes into the amplitude. In what follows we first consider the contribution of only the first term of the sum in eq.(11) and then discuss the effect of the higher terms. Keeping only the contribution of the first term in the sum in eq.(11) one can write the differential rate of the discussed decay in terms of the chromo-polarizability α_{1S} and the leptonic width $\Gamma_{ee}(1^3S1) \equiv \Gamma(1^3S_1 \to \ell^+\ell^-)$ in the form

$$d\Gamma(1^3S_1 \to \pi^+\pi^-\ell^+\ell^-) = \frac{[q^2 - C(q_0)]^2}{4b^2\, q_0^2} \, |\alpha_{1S}|^2 \sqrt{1 - \frac{4m_\pi^2}{q^2}} \, \sqrt{q_0^2 - q^2} \, \Gamma_{ee}(1^3S1)\, dq^2\, dq_0 \, , \tag{12}$$

where the formula (7) is used with a simplified parametrization of the subleading terms as a constant $C(q_0)$ similar to that in eq.(8).

In order to assess the feasibility of observing the discussed decays it can be noted that at a given constraint on the maximal value of q_0: $q_0 < \Delta$ (or equivalently at a lower cutoff on the invariant mass of the lepton pair) the probability described by eq.(12) strongly peaks near the highest values of both q^2 and q_0, i.e $q^2 \sim \Delta^2$ and $q_0 \sim \Delta$, and the total probability in the kinematical region constrained as $q_0 < \Delta$ scales approximately as Δ^6. However at higher q^2 both the dominance of the diagonal $1S - 1S$ transition in the sum in eq.(11) becomes weaker and the linear in q^2 behavior of the amplitude in eq.(7) derived for soft pions becomes questionable. It is still quite likely that with these limitations the presented here approach can be used up to somewhat higher values of Δ: $\Delta \approx 0.8 - 0.9\, GeV$, than those observed

in the pionic transitions from ψ' and Υ'. Indeed, experimentally the linearity in q^2 of the amplitude of the transition $\Upsilon' \to \pi\pi\,\Upsilon$ is very accurate[15, 13] in the physical region, i.e. up to $\sqrt{q^2} \approx 0.56\,GeV$ with no obvious hint of its violation close to this region. On the other hand the $f_0(980)$ resonance places a natural upper bound on the region of applicability of eq.(7). As to the contribution of higher quarkonium states in the sum in eq.(11), for each of these states the magnitude of this contribution relative to that of the diagonal transition is given by

$$r_n = \left| \frac{\alpha_{1S-nS}}{\alpha_{1S}} \, \frac{q_0}{M(n^3S_1) - M(1^3S_1) + q_0} \right| \, \left[\frac{\Gamma_{ee}(n^3S_1)}{\Gamma_{ee}(1^3S_1)} \right]^{1/2} \; . \tag{13}$$

The transition polarizability α_{1S-nS} should considerably decrease with n. This is supported by the very small experimental rate of the decay $\Upsilon(3S) \to \pi\pi\,\Upsilon$[1]. Thus, most likely, the only real effect of higher states up to $\Delta \sim 0.9\,GeV$ reduces to that of the 2S resonances. These however can be accounted for in the data analysis, since for these resonances all the parameters (except for the overall relative phase of their contribution) in eq.(13) are known. Furthermore an observation and analysis of the discussed decays at higher values of q_0, where the expression (7) is no longer valid, would be of a great interest for studies of the pion-pion scattering beyond the soft-pion region.

Numerically one can estimate from eq.(12) the total rate in the kinematical region constrained by $\Delta = 0.9\,GeV$ as

$$\Gamma(1^3S_1 \to \pi^+\pi^-\ell^+\ell^-)|_{q_0<0.9\,GeV} \approx 10^{-4} \left| \frac{\alpha_{1S}}{2\,GeV^{-3}} \right|^2 \Gamma_{ee}(1^3S_1) \; . \tag{14}$$

Given that the diagonal polarizability is likely to be larger than the transition one, it can be expected that for the J/ψ resonance the branching ratio of the discussed decay in a useable kinematical range should be at the level of 10^{-5} which looks to be well within the reach with the expected CLEO-c data sample. For the Υ resonances the effect is reduced by a factor of about 20 due to a smaller chromo-polarizability (cf. eq.(10)) and also due to a smaller value of $B(\Upsilon \to \ell^+\ell^-)$. Thus an experimental study of the discussed decay for the Υ resonance is likely to be a future task for a 10^{35} upgrade of KEKB[16] and/or the 10^{36} B-factory[17].

[1] The known problem with this decay is that the observed dipion mass spectrum does not agree with the formula in eq.(7) (see e.g. in the review [13]). A natural although quite qualitative explanation of this fact is that the polarizability α_{1S-3S} is very small, and subleading effects in the multipole expansion in QCD come into play in this transition.

Acknowledgments

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